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THE STRICTLY MIXED PROBLEM OF THE BENDING OF A THIN ELASTIC PLATE IN THE SHAPE OF A SEGMENT OF A CIRCLE†

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The strictly mixed problem [1] of the bending of a thin elastic plate in the shape of a segment of a circle under a uniform load is considered. Exact and approximate solutions of the problem together with an estimate of the root mean-square error are obtained. The behaviour of the bending moment at the corners and points where the boundary change is investigated.

1. STATEMENT OF THE PROBLEM

It is required to determine the deflection of a thin elastic sheet in the shape of a segment of a circle (see Fig. 1) bent by a uniform load of intensity q_0 = const such that the sheet is supported on the edge $0 < x < a, y = 0$ and clamped on the remaining edges.

In bipolar coordinates [2, p. 44], the problem reduces to solving the equation

$$\left[\frac{\partial^4}{\partial\alpha^4}+2\frac{\partial^4}{\partial\alpha^2\partial\beta^2}+\frac{\partial^4}{\partial\beta^4}+2\frac{\partial^2}{\partial\beta^2}-2\frac{\partial^2}{\partial\alpha^2}+1\right](w)=\frac{q_0h^3}{D} \tag{1.1}$$

$$(D = El^3/[12\ (1-\nu^2)],\ h = (a\ \mathrm{ch}a + \cos\beta)^{-1})$$

with boundary conditions

$$\begin{aligned} &w|_{\beta=\gamma} = w_\beta|_{\beta=\gamma} = 0,\ -\infty<\alpha<\infty \\ &w|_{\beta=0} = w_\beta|_{\beta=0} = 0,\ -\infty<\alpha<0 \\ &w|_{\beta=0} = w_{\beta\beta}|_{\beta=0} = 0,\ 0<\alpha<\infty \end{aligned} \tag{1.2}$$

Here l is the thickness of the sheet, E is Young's modulus and ν is Poisson's ratio.

2. REDUCTION OF THE PROBLEM TO A RIEMANN PROBLEM

We will first "extend" the boundary conditions [3, p. 175]

$$w|_{\beta=\gamma} = w_\beta|_{\beta=\gamma} = w|_{\beta=0} = 0,\ -\infty<\alpha<\infty \tag{2.1}$$

$$w_{\beta\beta}|_{\beta=0} = f_-(\alpha),\ w_\beta|_{\beta=0} = f_+(\alpha),\ -\infty<\alpha<\infty \tag{2.2}$$

Applying a Fourier transformation to boundary conditions (2.1)–(2.2), we obtain

$$\begin{aligned} &W(x,\ 0)=0,\ W_\beta(x,\ 0)=F^+(x),\ W_{\beta\beta}(x,\ 0)=F^-(x) \\ &W(x,\ \gamma)=0,\ W_\beta(x,\ \gamma)=0,\ -\infty<x<\infty \end{aligned} \tag{2.3}$$

Here

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$$W(x,\ \beta)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} w(\alpha,\ \beta)e^{ix\alpha}d\alpha$$

and $F^+(x)$ and $F^-(x)$ are respectively the limiting values of the analytic functions in the upper and lower half-planes.

Considering now the solution of Eq. (1.1), we represent $w(\alpha, \beta)$ in the form [2, p. 115]

$$w(\alpha,\ \beta)=\frac{q_0a^3}{16D}[w_0(\alpha,\ \beta)+(\operatorname{ch}\alpha+\cos\beta)^{-1}] \tag{2.4}$$

The second term in the square brackets is a particular solution of the inhomogeneous equation (1.1), and $w_0(\alpha, \beta)$ is the general solution of the corresponding homogeneous equation.

We will determine the Fourier transform $W(x, \beta)$ of the function $w_0(\alpha, \beta)$ and satisfy boundary conditions (2.3). This will yield the following Riemann problem

$$F^+(x)=A(x)(F^-(x)+\lambda H(x)),\ -\infty<x<\infty \tag{2.5}$$

Here

$$\frac{H(x)}{\sqrt{2\pi}}=\frac{x}{\operatorname{sh}\pi x}\left(\frac{2x^2-4}{3}+G_1(x)G_2(x)+4G_3(x)G_4(x)\right)+$$
$$+2G_3(x)G_5(x)+\frac{2\operatorname{sh}x\gamma}{\operatorname{sh}\pi x\sin\gamma}(G_1(x)G_4(x)-G_2(x)G_3(x)) \tag{2.6}$$

$$G_1(x)=\frac{x^2\sin 2\gamma-x\operatorname{sh}2x\gamma}{\operatorname{sh}^2 x\gamma-x^2\sin^2\gamma},\quad G_2(x)=\frac{x\operatorname{sh}2x\gamma+\sin 2\gamma}{\operatorname{ch}2x\gamma-\cos 2\gamma}$$

$$G_3(x)=\frac{x^2\operatorname{ch}x\gamma\sin\gamma-x\operatorname{sh}x\gamma\cos\gamma}{\operatorname{sh}^2 x\gamma-x^2\sin^2\gamma}$$

$$G_4(x)=\frac{\operatorname{ch}x\gamma\sin\gamma+x\operatorname{sh}x\gamma\cos\gamma}{\operatorname{ch}2x\gamma-\cos 2\gamma},\quad G_5(x)=\frac{x\operatorname{ch}x\gamma\sin\gamma-\operatorname{sh}x\gamma\cos\gamma}{\operatorname{sh}\pi x\sin^2\gamma}$$

$$A(x)=\frac{\operatorname{sh}^2 x\gamma-x^2\sin^2\gamma}{x(x\sin 2\gamma-\operatorname{sh}2x\gamma)},\quad \lambda=\frac{q_0a^3}{16D}$$

Note that we have used a representation of the Fourier transform of the function $w_0(\alpha, \beta)$ of [4] to obtain the Riemann problem (2.5).

3. THE EXACT SOLUTION OF THE RIEMANN PROBLEM

We will represent the Riemann problem (2.5) in the slightly different form

$$\sqrt{x+iq}^{\,+}K(x)F^+(x)=-\frac{F^-(x)}{2\sqrt{x-iq}^{\,-}}-\frac{\lambda H(x)}{\sqrt{x-iq}^{\,-}} \tag{3.1}$$

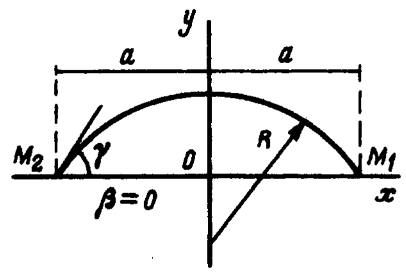

Fig. 1.

where

$$K(x)=-\left(2\sqrt{x^2+q^2}A(x)\right)^{-1},\ \lim_{|x|\to\infty}K(x)=1,\ \operatorname{Ind}K(x)=0 \tag{3.2}$$

Here $\sqrt{z}^+$ and $\sqrt{z}^-$ are defined and analytic in the upper and lower half-planes respectively, and their imaginary part is positive.

Starting from the structure (2.6) of the function $H(x)$, we will seek a solution of the Riemann problem (3.1) in the class of functions [3, p. 23] such that

$$\left(F^+(x)\sqrt{x+iq}^+\right)\in L_2^+,\ \left(F^-(x)\left(\sqrt{x-iq}^-\right)^{-1}\right)\in L_2^- \tag{3.3}$$

For solutions of this class, in view of condition (3.2), problem (3.1) has a unique solution [3, p. 23], which can be written in the form

$$F^+(x)=X^+(x)\left(\sqrt{x+iq}^+\right)^{-1}\Omega^+(x) \tag{3.4}$$

$$F^-(x)=-2X^-(x)\left(\sqrt{x-iq}^-\right)\Omega^-(x) \tag{3.5}$$

where

$$X^\pm(x)=\exp\left\{\frac{\operatorname{sgn}t\pm1}{2}V^{-1}(\ln K(\tau))(t)\right\}(x) \tag{3.6}$$

$$\Omega^\pm(x)=V\left\{\frac{\operatorname{sgn}t\pm1}{2}V^{-1}\left(\frac{H(\tau)}{X^-(\tau)\sqrt{\tau-iq}^-}\right)(t)\right\}(x) \tag{3.7}$$

Here V and V^{-1} are operators of the direct and inverse Fourier transformation, respectively.

Since it is difficult to solve the Riemann problem (3.4)–(3.7) numerically, we have the problem of obtaining an approximate solution of the Riemann problem (3.1) and, naturally, of the original problem with a corresponding error estimate.

4. AN APPROXIMATE SOLUTION OF THE RIEMANN PROBLEM. ERROR ESTIMATE.

In addition to the Riemann problem (2.5), we will consider the corresponding approximate problem

$$\sqrt{x+iq}^+\tilde{K}(x)\tilde{F}^+(x)=-\frac{\tilde{F}^-(x)}{2\sqrt{x-iq}^-}-\frac{\lambda\tilde{H}(x)}{\sqrt{x-iq}^-} \tag{4.1}$$

where

$$\tilde{K}(x)=-\left(2\sqrt{x^2+q^2}\tilde{A}(x)\right)^{-1},\ \tilde{A}(x)=-\frac{1}{2}(x^2+p^2)^{-1/2}\frac{x^2+a_1^2}{x^2+b_1^2} \tag{4.2}$$

$$\tilde{H}(x)=\sqrt{2\pi}(\tilde{H}_1(x)+\tilde{H}_2(x)) \tag{4.3}$$

$$\tilde{H}_1(x)=\frac{2x^2-4}{3}\frac{4a_2^2}{(x^2+1)(x^2+4)(x^2+a_2^2)} \tag{4.4}$$

$$\tilde{H}_2(x)=\frac{\tilde{H}_2(0)b_2^2}{(x^2+1)(x^2+b_2^2)},\ H_2(0)=\tilde{H}_2(0) \tag{4.5}$$

The parameters a_1, b_1, p, a_1, b_2, p are not known in advance, but are chosen such that the approximate solution has minimum error. The solution is found using the formulae

$$\tilde{F}^+(x)=\tilde{X}^+(x)\left(\sqrt{x+iq}^+\right)^{-1}\tilde{\Omega}^+(x) \tag{4.6}$$

$$\tilde{F}^{-}(x) = -2\tilde{X}^{-}(x)\left(\sqrt{x-iq}^{-}\right)\tilde{\Omega}^{-}(x) \tag{4.7}$$

$$\tilde{\Omega}^{\pm}(x) = V\left\{\frac{\operatorname{sgn} t \pm 1}{2} V^{-1}\left(\frac{\lambda\tilde{H}(\tau)}{\tilde{X}^{-}(\tau)\sqrt{\tau-iq}^{-}}\right)(t)\right\}(x) \tag{4.8}$$

The functions $\tilde{X}^{\pm}(x)$ are found by factorizing the coefficient $\tilde{K}(x)$. By putting $p = q$, we have the simple factorizations

$$\tilde{X}^{+}(x) = \frac{x+a_1 i}{x+b_1 i}, \quad \tilde{X}^{-}(x) = \frac{x-b_1 i}{x-a_1 i}$$

The estimate for the root mean-square error of the approximate solution is obtained as in [3, p. 156]. For (3.3), after further investigation, we obtain the error estimate

$$\left(\int_{-\infty}^{+\infty}\left|\Delta F^{+}(x)\sqrt{x+iq}^{+}\right|^{2} dx + \int_{-\infty}^{+\infty}\left|\Delta F^{-}(x)\sqrt{x-iq}^{-}\right|^{2} dx\right)^{1/2} \leqslant \varepsilon \tag{4.9}$$

$$(\Delta F^{\pm}(x) = F^{\pm}(x) - \tilde{F}^{\pm}(x))$$

Notice that

$$\|\Delta F^{+}(x)\|_2 \leqslant \frac{\varepsilon}{\sqrt{p}}\left(\|\varphi\|_2 = \left(\int_{-\infty}^{+\infty}|\varphi(x)|^2 dx\right)^{1/2}\right) \tag{4.10}$$

Here

$$\varepsilon = \frac{L^2}{1-\eta}\left(\varepsilon_1\|\tilde{F}^{+}\|_2 + \sqrt{\pi}\lambda\varepsilon_2\right), \quad L = \max\left(\frac{a_1}{b_1}, \frac{b_1}{a_1}\right), \quad \eta = \sqrt{2}L\varepsilon_1$$

$$\varepsilon_1 = \max_{-\infty<x<\infty}\left|\frac{1}{\tilde{A}(x)} - \frac{1}{A(x)}\right|\frac{1}{2\sqrt{p}}, \quad \varepsilon_2 = \max_{-\infty<x<\infty}\left|\sqrt{x^2+1}(H(x)-\tilde{H}(x))\right|.$$

For the approximate solution to have the minimum root mean-square error, the quantity ε must be as small as possible, and the function $\tilde{H}(x)$ must be as "close" as possible to the function $H(x)$. By minimizing these quantities we fix the parameters a_1, b_1, p, a_1, b_2. Different values of the angles γ characterizing the shape of the section will correspond to different parameters a_1, b_1, p, a_1, b_2 and quantities ε_1, ε_2. The values obtained for the parameters a_1, b_1, p, b_2, (a_2 = 1.4) and quantities ε_1, ε_2 for different values of γ are given below

γ	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3}{8}\pi$	$\frac{\pi}{2}$	$\frac{5}{8}\pi$	$\frac{3}{4}\pi$	$\frac{7}{8}\pi$	π
a_1	9.539	7.4983	2.8522	2.0602	1.6175	1.0433	0.7713	0.5975
b_1	7.1984	3.596	2.4004	1.8143	1.4804	0.8743	0.6427	0.5149
p	8.8517	3.822	2.1785	1.3802	0.9214	0.8052	0.6029	0.4285
ε_1	0.1553	0.1074	0.0834	0.0684	0.0595	0.0325	0.0245	0.0223
ε_2	1.065	1.075	1.155	1.286	1.51	1.5332	13.296	35.04
b_2	7.213	6.058	7.421	3.152	1.753	0.875	0.074	0.002

5. CONSTRUCTION OF AN APPROXIMATE SOLUTION OF THE RIEMANN PROBLEM IN EXPLICIT FORM

The functions defined by formula (4.8) form a component part of the approximate solution of the Riemann problem (4.1). Their direct use can lead to difficulties, which are relatively easily overcome by taking into account the structure of the function $\Omega(x)$. We will find the functions $\Omega^{\pm}(x)$ in succession. We will first solve the problem of a "jump" for the function

$$\tilde{G}(x)=\lambda\frac{x-a_1 i}{x-b_1 i}\tilde{H}(x)=\tilde{G}^+(x)-\tilde{G}^-(x)$$

where

$$\tilde{G}^+(x)=\sum_{k=1}^{4}\frac{iA_k}{x+\alpha_k i},\quad \alpha_1=1,\ \alpha_2=2,\ \alpha_3=a_2,\ \alpha_4=b_2$$
$$\tilde{G}^-(x)=\sum_{k=1}^{4}\frac{iB_k}{x-\alpha_k i}-\frac{iB_0}{x-b_1 i} \tag{5.1}$$

We will use the method of undetermined coefficients to find the constants A_k, B_k, B_0 and then solve the problem of a "jump" for the function

$$\tilde{\Omega}(x)=\left(\sqrt{x-ip}^-\right)^{-1}\tilde{G}(x)=\tilde{\Omega}^+(x)-\tilde{\Omega}^-(x)$$

Clearly

$$\tilde{\Omega}^+(x)=\sum_{k=1}^{4}\frac{iA_kC_k}{x+\alpha_k i}$$
$$\tilde{\Omega}^-(x)=\frac{\tilde{G}^-(x)}{\sqrt{x-ip}^-}+\sum_{k=1}^{4}\frac{iA_k\left(\left(\sqrt{x-ip}^-\right)^{-1}-C_k\right)}{x+\alpha_k i} \tag{5.2}$$

where

$$C_k=-2\sqrt{2}/(\sqrt{\alpha_k+p})$$

This gives an approximate solution of the Riemann problem (4.1) in explicit form. It can be constructed by applying formulae (5.1), (5.2) and (4.3)–(4.7) in succession. The relation between this solution and the angle γ will naturally agree with the results obtained in Section 4.

6. EXACT AND APPROXIMATE SOLUTIONS OF THE ORIGINAL PROBLEM. ERROR ESTIMATE

Solution of the Riemann problem (2.5) is only an intermediate stage in the solution of the original problem. The original problem is solved by applying an inverse Fourier transformation to the function $W(\alpha, \beta)$. The approximate solution $\tilde{W}(\alpha, \beta)$ is constructed with the same formulae as $W(\alpha, \beta)$, only with $F^+(x)$ replaced by $\tilde{F}^+(x)$

$$\left(w_\beta(x,\ 0)=F^+(x),\ \tilde{w}_\beta(x,\ 0)=\tilde{F}^+(x)\right).$$

The approximate solution $\tilde{w}(\alpha, \beta)$ of the original problem is obtained by applying the operator of the inverse Fourier transformation to the function $\tilde{W}(\alpha, \beta)$. The error estimate will be

$$\| w(\alpha,\beta)-\tilde{w}(\alpha,\beta)\|_2\leq M_1\|\Delta F^+(x)\|_2,\quad M_1=\max_{\substack{x\in R\\ \beta\in(0,\gamma)}}U_2(x,\beta) \tag{6.1}$$

(the function $U_2(x, \beta)$ appears in the representation of $w_0(x, \beta)$ [4]).

Taking into account inequality (4.11), we have the error estimate

$$\| w(\alpha,\beta)-\tilde{w}(\alpha,\beta)\|_2\leq\frac{M_1\varepsilon}{\sqrt{p}} \tag{6.2}$$

7. APPROXIMATE CALCULATION OF THE BENDING MOMENTS AT THE CLAMPED EDGE

The bending moment at the clamped edge is an important parameter of the theory of the bending of thin plates. Using the well-known formulae [2, p. 109], we have

$$\tilde{M}_0(\alpha) = -\frac{D}{a}(\operatorname{ch}\alpha + 1)V^{-1}\left(\left.\frac{d^2\tilde{W}(\alpha,\beta)}{d\beta^2}\right|_{\beta=0}\right) = -\frac{D}{a}(\operatorname{ch}\alpha + 1)\tilde{f}_-(\alpha) \tag{7.1}$$

$$\tilde{M}_\gamma(\alpha) = -\frac{D}{a}(\operatorname{ch}\alpha + 1)V^{-1}\left(\left.\frac{d^2\tilde{W}(\alpha,\beta)}{d\beta^2}\right|_{\beta=\gamma}\right) \tag{7.2}$$

Thus in order to determine the bending moment on the clamped part of the boundary $\beta = 0$, we must find $\tilde{f}_-(\alpha) = (V^{-1}\tilde{F}^-(x))(\alpha)$. Using the results of Section 5, we will find the function $\tilde{f}_-(\alpha)$ in explicit form

$$\tilde{f}_-(\alpha) = 2\sqrt{2\pi}\lambda\left(B_0 e^{a_1\alpha} + \sum_{k=1}^{4}\left(\frac{a_1 - b_1}{a_1 - \alpha_k}\right)B_k e^{a_1\alpha} + \sum_{k=1}^{4}\frac{\alpha_k - b_1}{\alpha_k - a_1}B_k e^{\alpha_k\alpha} - \right.$$

$$\left. - \sum_{k=1}^{4} A_k\left(1 + \frac{b_1 - \alpha_1}{a_1 + \alpha_k}\right)\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^{i\sqrt{\alpha_k + p}|\sqrt{\alpha}|} e^{\tau^2}\,d\tau\right)\right) +$$

$$+2\lambda(a_1 - b_1)(p - a_1)\sum_{k=1}^{4} A_k\sqrt{\frac{2}{\alpha_k + p}}\int_\alpha^0 e^{t(p-\alpha_k)}\frac{dt}{|\sqrt{t}|} - 4\sqrt{2}\lambda\sum_{k=1}^{4}\frac{A_k e^{p\alpha}}{\sqrt{\alpha_k + p}\ \sqrt{|\alpha|}}, \quad \alpha < 0 \tag{7.3}$$

To determine the bending moment on the clamped part of the boundary $\beta = \gamma$, we first find the function $d^2W/d\beta^2|_{\beta=\gamma}$. Using the representation (2.4), we find

$$\left.\frac{d^2\tilde{W}}{d\beta^2}\right|_{\beta=\gamma} = \lambda\left(\sum_{n=1}^{4} C_n(x)K_n(x) + K_5(x)\right) + \lambda C_3(x)(x^2 - 1) \tag{7.4}$$

where

$$\begin{aligned}
&\frac{C_1(x)}{\sqrt{2\pi}} = -\frac{x}{\operatorname{sh}\pi x}, \quad C_2(x) = \tilde{F}^+(x) \\
&\frac{C_3(x)}{\sqrt{2\pi}} = -\frac{\operatorname{sh}x\gamma}{\operatorname{sh}\pi x \cdot \sin\gamma}, \quad \frac{C_4(x)}{\sqrt{2\pi}} = -G_5(x) \\
&K_1(x) = 2(G_1(x)G_4(x) + G_2(x)G_3(x)), \quad K_2(x) = 2G_3(x) \\
&K_3(x) = 2(G_1(x)G_2(x) - G_3(x)G_4(x)), \quad K_4(x) = G_1(x) \\
&K_5(x) = \frac{(x^2 + 1)\operatorname{sh}x\gamma\sin^2\gamma - 2\cos\gamma(x\operatorname{ch}x\gamma\sin\gamma - \operatorname{sh}x\gamma\cos\gamma)}{\operatorname{sh}\pi x\sin^3\gamma}
\end{aligned} \tag{7.5}$$

Finally, we apply the inverse Fourier transformation to Eq. (7.5), using the theory of residues. The values of the roots of the transcendental equations

$$shz \pm z\,\sin 2\gamma/(2\gamma) = 0 \tag{7.6}$$

(cf. [2, p. 61, Tables 1 and 2]) will be useful here.

8. THE BEHAVIOUR OF THE BENDING MOMENTS AT CORNERS AND AT POINTS WHERE THE BOUNDARY CONDITIONS CHANGE

Since the approximate value of the moment $M_0(\alpha)$ has been found explicitly, it is possible to describe its behaviour at corners and at points where the boundary conditions change. We have the following asymptotic forms

$$\tilde{M}_0(\alpha) = O\left(\frac{1}{|\sqrt{\alpha}|}\right), \ \alpha \to -0, \ \tilde{M}_0(\alpha) \to \infty(\alpha \to -\infty, \ a_1 < 1)$$
$$\tilde{M}_0(\alpha) \to M_0 = 2\sqrt{2\pi}\lambda\left(\frac{b_1 - 1}{a_1 - 1}\right)B_1(\alpha \to -\infty, \ a_1 > 1, \ p > \alpha_k) \tag{8.1}$$

Use has been made of the properties of the probability integral and the fairly obvious limits

$$\lim_{\alpha \to -\infty} e^{-(\alpha_k+1)\alpha}\left(1 - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\alpha_k+p}|\sqrt{\alpha}|} e^{-\tau^2 d\tau}\right) = \begin{cases} 0, & p \geq 1 \\ \infty, & p < 1 \end{cases}$$

$$\lim_{\alpha \to -\infty} e^{(a_1-1)\alpha} \int_0^{\alpha} e^{t(p-\alpha_k)} \frac{dt}{\sqrt{|t|}} = \begin{cases} 0, & a_1 > 1, \ p > \alpha_k, \ k = 1,2,3 \\ 0, & a_1 > 1, \ p < \alpha_k, \ p - \alpha_k + a_1 - 1 \geq 0 \\ \infty, & a_1 < 1 \end{cases}$$

We will now investigate the behaviour of the moment $\tilde{M}_\gamma(\alpha)$ as $\alpha \to \pm\infty$. With this aim we write

$$\tilde{M}_\gamma(\alpha) = -\frac{q_0 a^2}{16}(\operatorname{ch}\alpha + \cos\gamma)\left(\sum_{n=1}^{4} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} c_n(t) k_n(t-\alpha)dt - c_3''(\alpha) - c_3(\alpha) + k_5(\alpha)\right) \tag{8.2}$$

$$c_n(t) = (V^{-1}C_n)(t), \ k_n(t) = (V^{-1}K_n)(t)$$

using the convolution theorem [3, p. 15]. Taking the limit in Eq. (8.2), we will have

$$\tilde{M}_\gamma(\pm\infty) = M_\gamma(\pm\infty) = M_\gamma =$$
$$= -\frac{q_0 a^2}{16}\left(\frac{2\gamma - \sin 2\gamma}{8\gamma \sin 2\gamma} \frac{\sin 2\gamma + \gamma\cos\gamma}{\gamma\sin 2\gamma - 2\sin^2\gamma} + \frac{2\gamma\cos\gamma - \sin 2\gamma}{4(\gamma\sin 2\gamma - 2\sin^2\gamma)} \frac{\sin 2\gamma + \gamma}{\gamma \sin 2\gamma} + \right.$$
$$+ \frac{\sin 2\gamma + 2\gamma}{2\gamma\sin 2\gamma} \frac{\sin 2\gamma - 2\gamma}{\gamma\sin 2\gamma - 2\sin^2\gamma} + \frac{\sin 2\gamma + 2\gamma\cos\gamma}{2\gamma\sin 2\gamma} \frac{\sin 2\gamma - 2\gamma\cos\gamma}{\gamma\sin 2\gamma - 2\sin^2\gamma} +$$
$$\left. + \frac{4\cos^2\gamma - \sin 2\gamma}{2\sin^3\gamma} + \left(\cos\gamma - \frac{1}{2}\right)\frac{\sin 2\gamma - 2\gamma\cos\gamma}{\gamma\sin 2\gamma - 2\sin^2\gamma} + 2\right) \tag{8.3}$$

(using relations (7.5) and the Tauber-type theorem of [5]). We have also assumed that $b_1 \geq 1$, $\alpha_k \geq 1$, $\forall k$ and $\gamma < \pi/2$. The last condition ensures that the functions $K_n(z)$ $(n = 1, 2, 3, 4)$ are analytic in the strip $|\operatorname{Im} z| \leq 1$, and therefore guarantees that the limits $\lim_{k \to i} K_n(z)$ exist.

9. CONCLUSIONS

All the limiting operations in Eq. (8.2) were taken for $\gamma < \pi/2$. Also, we used the limits

$$\lim_{\alpha \to +\infty} f_+(\alpha) = \lim_{\alpha \to +\infty} \tilde{f}_+(\alpha) = 0$$

When $\gamma \geq \pi/2$, the moment $M_\gamma(\alpha)$ tends to infinity. In that case, the first roots of Eqs (7.6) become as important as in simpler problems of this kind [2]. On the other hand, if $\gamma < \pi/2$, the moment at corners M_1 and M_2 (see Fig. 1) is non-zero and can be computed exactly for different γ using formula (8.3).

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